Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2009-2010
Semester Examination : Statistics (I)

Date: 25-11-2009 **Maximum Score 60** **Duration: 3 Hours.**

1. Crimes often go unreported. To model the number of instances of crimes sometimes two parameter binomial distribution is used. Let $X_1,\ X_2, ..., X_n$ be a random sample from *Binomial(m,θ),* where both m and θ are unknown. Obtain the *method of moments (mom) estimators* for m and θ.

[6]

2. Let $X_1,\ X_2, ..., X_n$ be a random sample from $f(x \mid \theta) = \theta x^{-2} e^{-\frac{x}{\theta}} I_{(0,\infty)}(x)$, where $\theta \in (0,\infty)$.

   (a) Check that f is indeed a *pdf.*

   (b) Show that the *maximum likelihood estimator* (*mle*) for θ is given by the *harmonic mean.*

[$2+6=8$]

3. Consider random variable Y with triangular distribution whose probability density function *(pdf)* is given by

$$f_Y(y) = \left| \begin{array}{ll} 4y & \text{if } 0 \le y \le \frac{1}{2} \\ 4-4y & \text{if } \frac{1}{2} < y \le 1 \\ 0 & \text{otherwise.} \end{array} \right.$$

   Assume that you can generate observations on $uniform\,[0,1]$.

   (a) How would you draw observations on Y using probability integral transform?

   (b) If U and V are independent $uniform\,[0,1]$ then obtain the distribution of $\frac{U+V}{2}$. Use this result to generate observations on Y.

   (c) How would you generate observations on Y using *Accept-Reject Algorithm.* Prove that your algorithm indeed generates observations on Y?

[$5+6+7=18$]

[PTO]

4. Ten observations on effective service life in minutes of batteries used in a laptop are as follows:

$$176, 191, 214, 220, 205, 192, 201, 190, 183, 185.$$

To investigate the hypothesis that battery life is adequately modelled by a normal distribution draw and interpret the probability plot.

[8]

5. A slot machine in Dan's casino charges \$2 per game. Let X be the net gain by a player on that slot machine in a game. The slot machine is supposed to yield the following distribution for X.

$X = x$: *net gain* (\$)	-2	23	48	73	98
$P(X = x)$	0.977	0.008	0.008	0.006	0.001

(1)

The slot machine yielded tidy profits to Dan until recently. Of late he got the impression that the machine was yielding the jackpot ($X = 98$) a bit too often. He, therefore collected data on 1000 randomly selected games on the slot machine and observed the following frequencies of net gains made by the players.

x : *net gain* (\$)	-2	23	48	73	98	*Total*
Observed frequency	965	10	9	9	7	1000

Do the data support the hypothesis that they actually come from the distribution specified in (1)?

[12]

6. Aircrew escape systems are powered by a solid propellent. The burning rate of this propellent is an important product characteristic. Specifications require that the mean burning rate must be 50 $cm/\sec$. We know that the standard deviation of burning rate is $\sigma = 2\ cm/\sec$. The experimenter decides to specify level of significance to be $\alpha = 0.05$. She selects a random sample of $n = 25$ and obtains a sample average burning rate of $\overline{x} = 51.3\ cm/\sec$.

   (a) What conclusions should she draw?

   (b) Also obtain $p - value$.

   (c) Obtain 90% confidence interval for the mean burning rate.

[$8 + 2 + 4 = 14$]